Technical Notes

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Migration of Solid Particles Perpendicular to a Local Shear Flow Due to Local Instabilities

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Introduction

THIS investigation considers a flowfield generated by the superposition of a shear flow in the x direction and a perturbation in the form of a small oscillatory flow in the y direction. See Fig. 1. A small particle of radius a is assumed to be placed in the flow and to be affected by it.

Because of the oscillatory motion, the particle may have a velocity component perpendicular to the shear flow. The entire state, i.e., the particle with the flowfield, is denoted stable if the particle maintains, on the average, its y coordinate. If the particle leaves its initial average y coordinate and does not return, the state is denoted unstable. Conditions are sought to determine the stability or instability of a given state.

The particle is assumed small and, therefore, the shear flow need be only a local shear flow. Thus, the results obtained here are quite general, because many flows can be locally approximated by shear flows.

The motivation for this investigation stems from the need to predict the distribution of particles and their migration characteristics in turbulent flows, e.g., in air pollution considerations, in dusting fields, in rivers, and in the use of particles to measure velocities in laser techniques.

The analysis is done in detail for perturbations in the y direction. It is noted, however, that the same results are obtained for perturbations in the x direction, as indicated briefly in the analysis.

Analysis

Instability Conditions for a Rigid Particle in a Flow

The velocity field is given by

$$q = \hat{i}U + \hat{j}V, \qquad U = hy$$
$$V = \epsilon \cdot V(y)\exp[i(\omega t + kx)] \tag{1}$$

with $\epsilon \ll 1$.

The equations for the motion of particles in this flowfield are

$$\frac{d^2x}{dt^2} = A\left(U - \frac{dx}{dt}\right)$$
$$\frac{d^2y}{dt^2} = A\left(V - \frac{dy}{dt}\right) + L\left(U - \frac{dx}{dt}\right) \tag{2}$$

The coefficients in Eq. (2), which relate to drag[1] A and to lift[2-4] L are given by

$$A = \frac{6\pi\mu a}{m_p} = \frac{9\mu}{2a^2\rho_p}$$
$$L = L_1 + L_2$$
$$L_1 = \frac{6.46 \cdot \mu \cdot a^2}{\nu^{1/2} m_p}\left(\frac{dU}{dy}\right)^{1/2} \approx \frac{1.54\mu h^{1/2}}{\nu^{1/2} a \cdot \rho_p}$$
$$L_2 = \frac{\pi\Omega a^3 \rho_f}{m_p} = \frac{3}{4}\,\Omega\,\frac{\rho_f}{\rho_p} \tag{3}$$

where a is the particle diameter, m_p its mass, ρ_p its density, Ω its angular velocity in the x-y plane, μ the dynamic viscosity of the fluid, and ν its kinematic viscosity.

A necessary condition for Eqs. (3) to hold is that all three Reynolds numbers

$$R_h = \frac{ha^2}{\nu}, \qquad R_v = \frac{va}{\nu}, \qquad R_\Omega = \frac{\Omega a^2}{\nu}$$

are small compared with 1.

Equations (1) and (2) yield

$$\frac{d^2x}{dt^2} = A\left(hy - \frac{dx}{dt}\right)$$
$$\frac{d^2y}{dt^2} = A\left[\epsilon v(y)e^{i(\omega t + kx)} - \frac{dy}{dt}\right] + L\left(hy - \frac{dx}{dt}\right) \tag{4}$$

with the initial conditions

$$y = y_{in}, \quad x = 0, \quad \frac{dx}{dt} = U = hy, \quad \text{at } t = 0 \tag{5}$$

The oscillatory motion is small and a solution is sought as a series expansion in ϵ,

$$x = x_0 + \epsilon x_1 + \ldots$$
$$y = y_0 + \epsilon y_1 + \ldots \tag{6}$$

The zeroth approximation to Eqs. (4) is

$$\frac{dx_0}{dt} = hy_0, \quad \frac{dy_0}{dt} = 0 \tag{7}$$

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with the initial condition

$$y_0 = y_{in}, \quad x_0 = 0, \qquad \text{at } t=0 \tag{8}$$

The zeroth approximation to the solution is

$$y_0 = y_{in}, \quad x_0 = hy_0 t \tag{9}$$

The first-order approximation of Eqs. (4) with Eq. (9) yield

$$\frac{d^2x_1}{dt^2} + A\frac{dx_1}{dt} - Ahy_1 = 0 \tag{10a}$$

$$\frac{d^2y_1}{dt^2} + A\frac{dy_1}{dt} - Lhy_1 + L\frac{dx_1}{dt} = AV_0 e^{i(\omega t + khy_0 t)} \tag{10b}$$

where $V_0 = V(y_0)$.

Equation (10b) is rearranged

$$\frac{dx_1}{dt} = -\frac{1}{L}\frac{d^2y_1}{dt^2} - \frac{A}{L}\frac{dy_1}{dt} + hy_1 + AV_0 e^{i\bar{\omega}t}$$

and differentiated

$$\frac{d^2x_1}{dt^2} = -\frac{1}{L}\frac{d^3y_1}{dt^3} - \frac{A}{L}\frac{d^2y_1}{dt^2} + h\frac{dy_1}{dt} + i\bar{\omega}AV_0 e^{i\bar{\omega}t}$$

where $\bar{\omega} \equiv \omega + khy_0$.

Substitution into Eq. (10a) yields

$$\frac{d^3y_1}{dt^3} + 2A\frac{d^2y_1}{dt^2} + (A^2 - Lh)\frac{dy_1}{dt} = i\bar{\omega}AV_0 e^{i\bar{\omega}t} + A^2V_0 e^{i\bar{\omega}t} \tag{11}$$

The characteristic equation of the homogeneous part of Eq. (11) is

$$\lambda^3 + 2A\lambda^2 + (A^2 - Lh)\lambda = 0 \tag{12}$$

with the solution

$$\lambda_1 = 0, \quad \lambda_{2,3} = -A \pm \sqrt{Lh} \tag{13}$$

It is noted that if, instead of y oscillations, there are x ones, the resulting equation for y_1 will differ from Eq. (11) in its right-hand side only and the characteristic equation for the λ will still be the same. Thus, the stability conditions, which result only from the characteristic equation for λ, are quite general and apply to both x and y perturbations.

The solution for y_1 is unstable for

$$-A + \sqrt{Lh} > 0$$

or

$$\sqrt{Lh}/A > 1 \tag{14}$$

Using Eq. (3), Eq. (14) becomes

$$\frac{Lh}{A^2} = \frac{1.54\mu h^{3/2}}{\nu^{1/2}a\rho_p(81/4)(\mu^2/a^4\rho_p^2)} = \frac{0.076h^{3/2}a^3\rho_p}{\nu^{1/2}\mu} > 1$$

Therefore, the condition for instability is

$$a^3 > 13.12\frac{\nu^{1/2}\mu}{h^{3/2}\rho_p} = 13.2\left(\frac{\nu}{h}\right)^{3/2}\frac{\rho_f}{\rho_p} \tag{15}$$

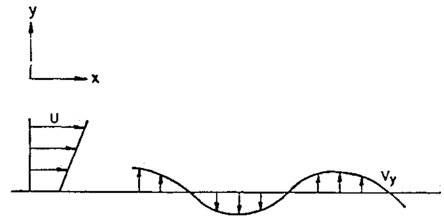

Fig. 1 Velocity field and coordinates.

Equation (15) contains neither the magnitude of the perturbation nor its frequency and, therefore, the instability does not depend on these. Thus, when Eq. (15) holds, a small perturbation in the motion of a particle moving in shear flow will cause it to migrate with increasing velocity in the direction perpendicular to that of the main flow.

The same instability condition in Eqs. (14) and (15) holds in the case of a single shear flow with a small particle having an initial velocity perpendicular to the direction of the flow. This case is very important in the theory of the deposition of small particles from turbulent flow in tubes. A basic concept in the models of particle deposition[5,6] is that of the "stopping distance" of a small particle in the viscous sublayer, which can be considered as a simple shear flow. The "stopping distance" is the distance that a particle moving with an initial velocity in the direction perpendicular to the main flow covers before it stops. This distance has been calculated until now without taking into account the lift force.

Applying the present results and using the "friction velocity" $U^* = \sqrt{\tau_w/\rho_f}$ as the characteristic velocity of the flow in the viscous sublayer and the length $y^* = \nu/U_*$ as characteristic, the instability condition of Eq. (14) becomes

$$\tau^+ > 1/\sqrt{L^+h^+} \text{ with } h^+ = 1 \text{ in the sublayer} \tag{16}$$

where $\tau^+ = 1/A^+ = (2/9)(\rho_p/\rho_f)(aV_*/\nu)^2$ is the nondimensional particle relaxation time.

Thus, when Eq. (16) holds, a particle with an initial velocity in the direction perpendicular to the shear flow does not stop and continues its motion with increasing velocity until it hits the wall. If one insists on still using the concept of "stopping distance" in such cases, then the "stopping distance" becomes "until it hits the wall." These cases are very important in considerations of deposition in turbulent flows. For example, for $\rho_p = 1$ g/cm^2 and $\rho_f = 10^{-3}$ g/cm^3, Eq. (16) requires $\tau + \geq 12$. This value is in the middle of the range of interest in the theory of deposition of small particles in turbulent flows in tubes.

Conclusion

A criterion to the onset of migration of particles perpendicular to a local shear flow has been obtained. The form of the criterion is an algebraic relation between the diameter of the particle, the local shear rate, the kinematic viscosity of the fluid, and the ratio of the fluid/particle densitities, which can be put as

$$a\left(\frac{h}{\nu}\right)^{1/2}\left(\frac{\rho_p}{\rho_f}\right)^{1/3} \geq 2.36 \tag{17}$$

In laminar flows, regions in which the criterion for migration is satisfied are therefore cleared of particles whose size satisfies the criterion. Thus, qualitatively, in a laminar pipe flow one should find larger particles in the center only, intermediate size particles both in the middle and further toward the walls, and smaller particles everywhere.

An application of the criterion to the viscous sublayer in turbulent flows has resulted in the "stopping distance" for any particle that satisfies the criterion to be unlimited, i.e., once the particle enters the viscous sublayer it continues to move until it hits the wall.

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Computation of Unsteady Transonic Aerodynamics with Truncation Error Injection

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Introduction

THE urgent need for effective, reliable methods for unsteady aerodynamic predictions at transonic Mach numbers is evident from the Farmer and Hanson[1] experiment in which it was observed that the flutter boundary for a wing with a supercritical cross section is substantially lower than that with a conventional one. At transonic speeds, the size and location of the embedded supersonic zone over the wing affect the way acoustic signals propagate and, hence, the aerodynamic responses to disturbances. Recent developments in computational fluid dynamics and the availability of supercomputers have made accurate flow prediction possible. However, for applications like routine flutter calculation and aircraft design optimization, the currently available codes, especially the ones for three-dimensional computations like XTRAN3S of Rizzetta and Borland[2] and USTF3 of Isogai and Suetsugu,[3] are still much too time consuming.

As mentioned in Fung,[4] one of the problems in unsteady transonic flow computation is the grid for obtaining the solution. Aside from the issue of finding the best grid for a given wing geometry, a grid must have a local mesh size comparable to the radius of curvature of the leading edge in order to properly resolve the fast expansion that determines the size of the sonic bubble and the strength and location of the shock. The computational domain must be large enough to allow the flow to relax to the freestream condition with little confinement from grid boundaries. Almost all grids currently used for aerodynamic computations are based on these considerations. However, these grids, while suitable for computing steady flows, may require (due to linear or nonlinear numerical instability) too small a time-step limitation for efficient computations of the unsteady acoustic waves due to the small unsteady wing motions and deformations assumed in flutter analysis. For low-to-moderate reduced frequencies, the typical wavelength of an acoustic signal in a transonic flow is of the order of the chord of the wing. Hence, a grid with a minimum spacing of a tenth of the chord should be sufficient. However, an accurate prediction of the steady flowfield over a wing at supercritical Mach number often requires a minimum spacing of a hundredth of the chord and, hence, a time-step requirement based on the CFL condition 10 times as restricted as that needed for accuracy.

In this Note, a technique is introduced that allows the steady and unsteady flows to be computed on different grids. To demonstrate the efficiency of this technique, the unsteady small-disturbance transonic equation is used for unsteady aerodynamic prediction. The results of applying this technique are compared to those obtained on single grids.

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Computations with Truncation Error Injection

It has been shown[5] that, by solving the corresponding difference equation with the truncation error included as a forcing term, exact nodal values of the solution to a differential equation can be obtained; that if the exact solution were known, the exact truncation error can be computed at nodal points; and that the truncation error can be approximated by local grid refinement.

Consider a difference equation of the form

$$L_t\phi_h + L_h\phi_h = 0 \tag{1}$$

where L_t and L_h correspond to the temporal and spatial discrete operators, respectively, and ϕ_h the numerical solution on a grid of size h. Assuming that ϕ_h^0 is the steady-state solution of Eq. (1) satisfying

$$L_h\phi_h^0 = 0 \tag{2}$$

it is quite obvious that ϕ_h also satisfies

$$L_t\phi_h + L_h\phi_h = L_h\phi_h^0 \tag{3}$$

and that solving Eq. (3) for ϕ_h is the same as solving Eq. (1). However, Eq. (3) is more general in the sense that, if it converges, it will yield the steady state ϕ_h^0 regardless of whether ϕ_h^0 satisfies Eq. (2). For example, we could replace ϕ_h^0 by $\phi_{h/N}^0$, i.e.,

$$L_t\phi_h + L_h\phi_h = L_n\phi_{h/N}^0 \tag{4}$$

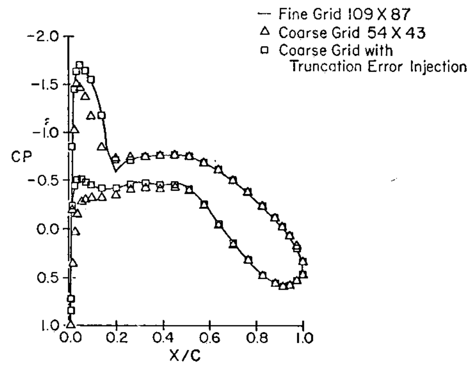

Fig. 1 Comparison of the surface pressure distributions of an NLR 7301 airfoil at $M_\infty = 0.70$ obtained by different methods on different grids.